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50. Internationales Wissenschaftliches Kolloquium
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P. Flores / J.C.P. Claro / J. Ambrósio / H.M. Lankarani

Numerical and Experimental Study of Planar Mechanisms with Clearance Joints

ABSTRACT

A comprehensive numerical and experimental study on the dynamic response of a slider-crank mechanism with clearance joints is presented and discussed in this work. The main objective of this study is to provide an experimental verification and validation of the predictive capabilities of the clearance joint models, also proposed in this work. The experimental procedure complements the theoretical or numerical studies in the literature and it provides a proposal for the coherent combination of numerical and experimental work that needs to be undertaken, in order to establish validated models and to identify directions for subsequent studies that will allow for the improvement of the models. This study is supported in an experimental test rig that consists of a slider-crank mechanism with an adjustable radial clearance at the revolute joint between the slider and the connecting rod. The motion of the slider is measured with a linear transducer and an accelerometer. Dynamic tests at different operating crank speeds and with several clearances are performed. The maximum slider acceleration, associated with the impact acceleration, is used as a measure of the impact severity. The results obtained demonstrate the dynamical behavior of a clearance joint and they provide qualitative measures that can be associated with fatigue and wear phenomena, when the system components have to operate with real joints. Furthermore, the correlation between the numerical and experimental results is presented and discussed.

INTRODUCTION

A mechanism is made of several components, which can be divided in two major groups, namely, links, that is, bodies with a convenient geometry, and joints, which introduce some restrictions on the relative motion of the various bodies of the system. Usually, the bodies are modeled as rigid and/or deformable bodies, while the joints are modeled through a set of kinematic constraints, that is, the joints are not modeled as contact pairs in the stick sense of the word contact but as algebraic constraints to which implicit forces are associated. The functionality of a joint relies upon the relative motion allowed between the connected components. In most cases, this implies the existence of a clearance between the mating parts, and thus surface contact, shock transmission and the development of different regimes of friction and wear. On the other hand, no matter how small that clearance is, it can lead to vibration and fatigue phenomena, lack of precision or, even random overall behavior.

Over the last few decades, a number of researchers have proposed various methodologies for modeling mechanisms with clearance joints [1-4]. However, most of these studies only deal with numerical models. The literature reporting on experimental studies on mechanical systems with clearance joints is confined to a few publications. Dubowsky and Moening [5] studied experimentally the interactions between clearance joints and the system elasticity, using a Scotch--Yoke mechanism. They applied accelerometers to measure the impact accelerations. A failure occurred at the Scotch-Yoke mechanism due to fatigue, caused by large impact forces developed at the clearance joint. Grant and Fawcett [6] investigated the effects of clearance size, lubrication and material properties on the contact loss in a four bar linkage with one clearance joint. Their experimental results confirmed the validity of the theoretical approach proposed, but only for a limited class of systems. Dubowsky et al. [7] studied analytically and experimentally a simple system, called the Impact Ring Model, to predict the impacts in planar mechanical systems with clearance joints. Haines [8] carried out an experimental investigation on the dynamic behavior of a simple journal-bearing with varying degrees of freedom. In that study, the contact loss between the journal and bearing was predicted using proximity transducers. Bengisu et al. [9] studied the contact loss in revolute clearance joints of a four bar mechanism. The relative motion between the journal and bearing was measured by employing an optical method. Soong and Thompson [10] presented a theoretical and experimental investigation of the dynamic response of a slider-crank mechanism with a revolute clearance joint where the slider and the connecting rod accelerations were quantified by using accelerometers.

The main goal of this work is to provide an experimental verification and validation of the predictive capabilities of the clearance joint models revised in this work. The experimental procedure complements the numerical studies in the literature and it provides a proposal for the coherent combination of numerical and experimental work that needs to be undertaken, in order to establish validated models and to identify directions for subsequent studies that will allow for the improvement of the models. This study is supported in an experimental test rig that consists of a slider-crank mechanism with an adjustable radial clearance at the revolute joint between the slider and the connecting rod. Dynamic tests at different operating crank speeds and with several clearances are performed. The maximum slider acceleration, associated with the impact acceleration, is used as a measure of the impact severity. The results obtained demonstrate the dynamical behavior of a clearance joint and they provide qualitative measures that can be associated with fatigue and wear phenomena, when the system components have to operate with real joints. The correlation between the numerical and experimental results is presented and discussed.

MODELING REVOLUTE JOINTS WITH CLEARANCE

In the classical analysis of a revolute joint the journal and bearing centers coincide, that is, the revolute joint is considered ideal or perfect, but the inclusion of the clearance separates these two centers. Consequently, two extra degrees of freedom are added to the system. Figure 1a depicts a revolute clearance joint, the so-called journal-bearing, in which the radial clearance, c, is measured by the difference between the bearing and journal radius, R_B and R_J, respectively. A revolute clearance joint does not impose any kinematic constraints on the system, but limits the journal orbit inside the bearing's boundaries.

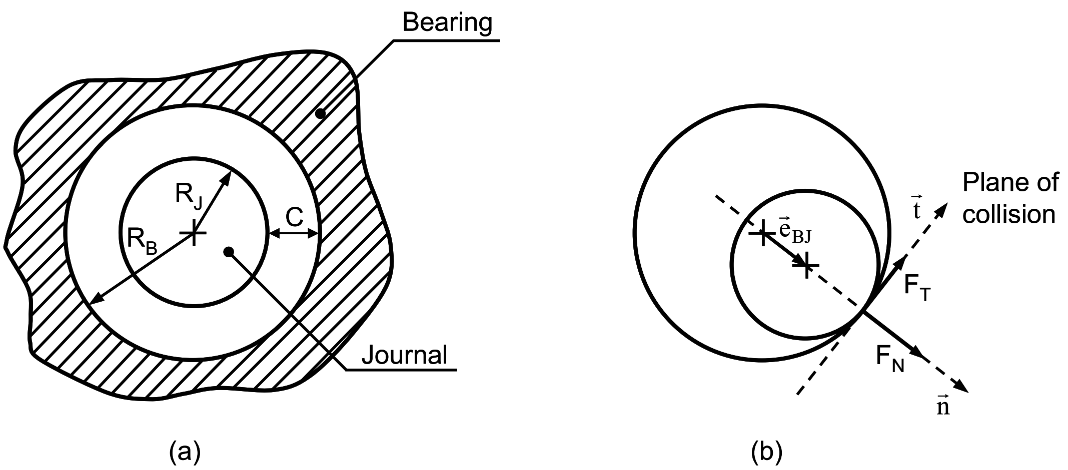


Figure 1 – (a) Revolute joint with clearance; (b) Normal and tangential forces due to the impact between the journal and bearing.

In a dry contact situation, the journal can move freely within the bearing until contact between the two bodies takes place. When the journal impacts the bearing wall, a normal contact force together with a friction law is evaluated to obtain the dynamics of the journal-bearing. These forces are of a complex nature, and their corresponding impulse is transmitted throughout the mechanical system. Figure 1b shows the normal and tangential force components due to the impact between the journal and the bearing. The impact which has both normal and tangential relative velocities is treated as an eccentric oblique collision between two bodies.

From dynamic configuration of the system, the relative penetration depth between the journal and the bearing can be defined as,

$$\delta = e_{BJ} - c \tag{1}$$

where e_{BJ} is the magnitude of the eccentricity vector defined between the bearing and journal center, and c is the radial clearance, which is a specified parameter. A good treatise on the revolute clearance joint modelization is presented by Flores et al. [11].

The dynamics of a dry journal-bearing is characterized by two different situations. Firstly, when the journal and bearing are not in contact each other, there is no contact forces associated to the journal-bearing. Secondly, when the contact between the two bodies takes place, the contact-impact forces are modeled according to a nonlinear Hertz force law (normal force) together with the Coulomb friction law (tangential force). These conditions can be expressed as,

$$\begin{aligned} \mathbf{f} &= 0 \qquad if \quad \delta < 0 \\ \mathbf{f} &= \mathbf{f}_N + \mathbf{f}_T \quad if \quad \delta > 0 \end{aligned} \tag{2}$$

in which, $\mathbf{f}_N$ and $\mathbf{f}_T$ are normal and tangential forces.

In short, when the journal reaches the bearing wall an impact takes place. This impact is treated as a continuous event, that is, the local deformations and the contact forces are continuous functions of time. The impact analysis of the system is performed simply by including the normal and tangential contact forces into the system's equations of motion.

For a revolute clearance joint, the contact between the journal and bearing can be modeled by using the Lankarani and Nikravesh contact force model given by [12],

$$F_N = K\delta^n \left[1 + \frac{3(1-e^2)}{4} \frac{\dot{\delta}}{\dot{\delta}^{(-)}} \right] \tag{3}$$

where K is the contact stiffness and δ is the relative penetration depth, e is the restitution coefficient, $\dot{\delta}$ is the relative penetration velocity and $\dot{\delta}^{(-)}$ is the initial impact velocity. The exponent n is set to 1.5 for metals.

The parameter K depends on the material and geometric properties of the contacting surfaces. For two spherical surfaces in contact, the generalized stiffness parameter is given by [13],

$$K = \frac{4}{3(h_B + h_J)} \left[\frac{R_B R_J}{R_B - R_J} \right]^{\frac{1}{2}} \tag{4}$$

where the parameters h_B and h_J are given by,

$$h_k = \frac{1 - \nu_k^2}{E_k}, \quad (k=B,J) \tag{5}$$

R_B and R_J are the radius of the bearing and journal, respectively, ν_k is the Poisson's ratio and E_k is the Young's modulus.

In a revolute clearance joint three different modes of motion between the journal and bearing can be considered, namely: contact or following mode, free-flight mode, and impact mode. These three types of the journal motion are illustrated in figure 2 [11].

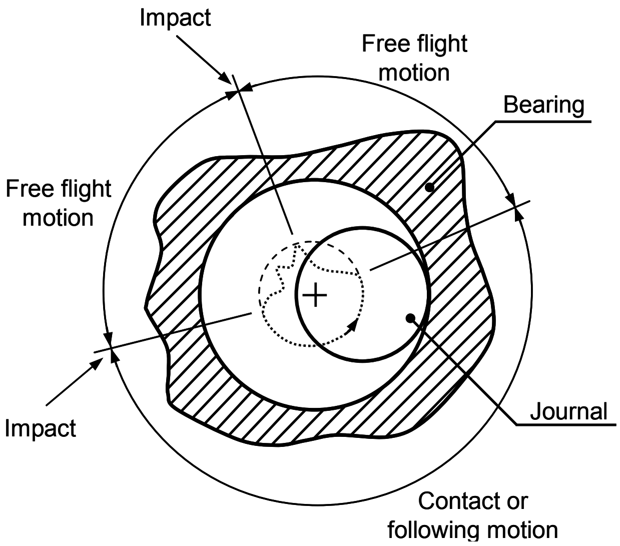


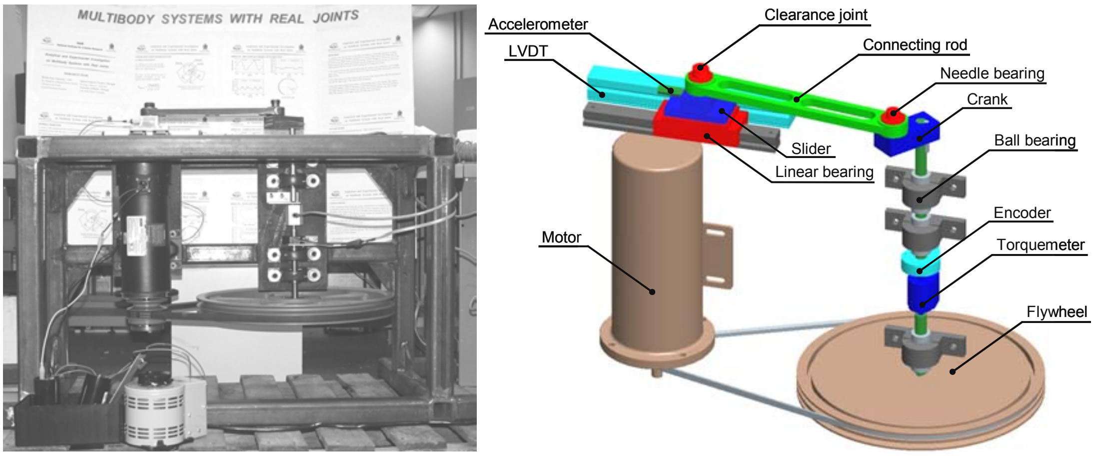
Figure 2 – Different types of journal motion inside the bearing.

In the contact or following mode, the journal and the bearing are in contact and a sliding motion related to each other is assumed to exist. In this mode the penetration depth varies along the circumference of the journal. Clearly in practice this mode is ended at the instant when the journal and bearing separate and the journal enters the free flight mode. In the free flight mode, the journal can move freely inside the bearing boundaries, i.e., the journal and the bearing joint are not in contact, hence there is no reaction force between these two elements. After the end of the free flight mode, the journal enters the impact mode. In the impact mode, which occurs at the termination of the free flight mode, impact forces are applied and removed. This mode is characterized by a discontinuity in the kinematic and dynamic characteristics, and a significant exchange of momentum occurs between the two impacting bodies. At the termination of the impact mode, the journal can enter either free flight or following mode. During the dynamic simulation of a revolute joint with clearance, if the path of the journal center was plotted for each instant (integration step), the different modes of motion of the journal inside the bearing can easily be observed.

EXPERIMENTAL TEST RIG

In order to investigate the dynamic response of mechanical systems with clearance joints, an experimental test rig was designed and constructed. The experimental equipment was designed with the intent of providing data that can support the identification of different models and the validation of the numerical models developed for dynamic analysis of mechanisms with clearance joints. The experimental test rig has been constructed and operated at the Computational Mechanics Laboratory at the National Institute for Aviation Research, Wichita State University, Kansas, USA.

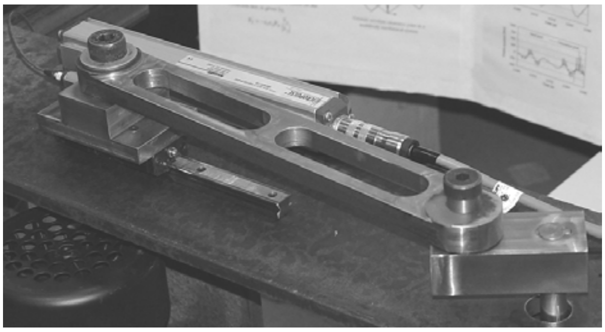
A slider-crank mechanism in which the revolute joint that connects the slider and connecting rod has a variable and controlled radial clearance was chosen, due to its simplicity and importance within all possible candidate machines and mechanisms. An overall view of the experimental apparatus built is shown in a photograph in Figure 3a, and in a schematic drawing in Figure 3b.

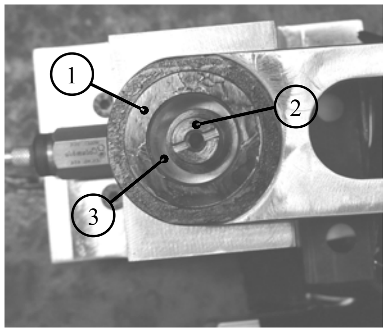


(a) (b)

Figure 3 – (a) Photograph of the experimental test rig; (b) Schematic drawing of the test rig.

The main sub-assembly of the experimental test rig consists of a slider-crank mechanism with an adjustable radial clearance at the revolute joint between the slider and the connecting rod, shown in Figure 4. This joint is designed as a dry journal-bearing, as illustrated in Figure 4b. The remaining kinematic joints were constructed as close to ideal as possible, that is, with minimum clearance and friction in order to minimize any contamination of the data that are intended to be measured. Moreover, these joints are lightly oiled to minimize the friction in the connections. A standard sleeve element was press-fitted to the extremity of the connecting rod, working as bearing, being its diameter fixed to a very tight tolerance. The journal is rigidly connected to the sliding block and incorporates a standard pin with a variable diameter, as pictured in Figure 4b. Thus, the clearance at the test journal-bearing can be altered by simply changing the pin. A particular journal-bearing set is also manufactured in order to simulate an ideal or zero-clearance joint, which is used to obtain the reference data associated with an ideal mechanism. Different sets of journal diameters, which together with the bearing diameter result in exaggerated radial clearances, are also built in order to allow the analysis of the influence of the clearance on the system's dynamic response. The crankshaft is keyed to the crank and it is supported by ball bearings. A needle bearing with a minimal radial clearance and high rigidity connects the crank to the connecting rod. The sliding block component is screwed onto a linear translational bearing, which has a precision preloaded system with zero-clearances. Table 1 shows the type of joints used in the experimental slider-crank mechanism and their nominal or operating clearances. Thus, for numerical purpose, they can be considered as ideal or zero-clearance joints.





(a) (b)

Figure 4 – (a) Experimental slider-crank mechanism with an adjustable radial clearance at the revolute joint between the slider and the connecting rod; (b) Test revolute joint in which the clearance is altered by simply changing the journal diameter: 1-bearing/sleeve, 2-journal/pin; 3-clearance. Clearance is exaggerated for illustration.

Connection	Joint type	Diameter [mm]	Clearance [mm]
Ground – Crank	Ball bearing	17.0	0.009
Crank – Connecting rod	Needle bearing	10.0	0.005
Connecting rod – Slider	Journal-bearing	22.2	0.002
Ground – Slider	Translational bearing	–	0.001

Table 1 – Type of joints used in the experimental slider-crank mechanism and the corresponding nominal clearances.

Between the driven pulley and the crankshaft an encoder and a torque sensor are incorporated. The encoder is used to measure the crank angular position and velocity, whereas the torque sensor allows the measurement of the reaction moment that acts on the crank. Furthermore, an accelerometer and a linear voltage differential transducer (LVDT) are used to monitor the slider acceleration and displacement, respectively. The slider velocity is obtained either by doing the numerical integration of the acceleration value, or the numerical differentiation of the displacement data. The impact force between the journal and bearing can be measured indirectly, that is, the impact accelerations are directly related to the impact forces.

The slider-crank mechanism works on the horizontal plane and, due to its rigidity and alignment, the gravitational effects on the system's dynamic responses can be neglected. The mechanism components are constructed entirely from steel and, hence for practical purposes, are assumed to be perfectly rigid. The connecting rod is constructed in a hollow form in order to reduce the mass maintaining a high stiffness. The slider-crank mechanism and all other mechanical components are mounted on a heavy stiff frame. The mechanical arrangement of the experimental test rig is schematically illustrated in Figure 3b. A summary of the physical properties of the experimental slider-crank model is given in Table 2 where the crank inertia properties include the shaft, encoder, torque sensor and flywheel. Similarly, the slider-block inertia properties take into account the linear bearing and the accelerometer characteristics. These values are used in the numerical simulations. The overall mass of the experimental equipment, including frame and moving parts, is about 130Kg.

Body	Length [m]	Mass [Kg]	Moment of inertia [Kgm^2]
Crank	0.05	17.900	0.460327
Connecting rod	0.30	1.130	0.015300
Sliding block	-	1.013	0.000772

Table 2 – Physical properties of the experimental slider-crank mechanism.

The experimental test rig allows to adjust two parameters: the journal diameter, i.e., the radial clearance, and the driving frequency, i.e., the crank speed. The radial clearance is obtained by changing the journal diameter and maintaining constant the diameter of the sleeve/bearing.

CORRELATION BETWEEN NUMERICAL AND EXPERIMENTAL RESULTS

In what follows, the experimental and numerical simulation results are correlated and discussed. The length and inertia properties of the experimental slider-crank mechanism components are listed in Table 2. The parameters used in the dynamic simulations are given in Table 3. Figures 5a-h and 6a-h present the numerical and experimental dynamic acceleration response histories of the slider-crank model with a clearance joint.

Restitution coefficient	0.46	Baumgarte - α	5
Friction coefficient	0.01	Baumgarte - β	5
Young's modulus	207 GPa	Integration step size	10^{-6} s
Poisson's ratio	0.3	Integration tolerance	10^{-7}

Table 3 – Simulation parameters for the experimental slider-crank mechanism.

In the modeling of the impact phenomenon in mechanical systems, the selection of the friction and restitution coefficients is of great importance and influences the outcome of the results. The selection of these parameters is made with the materials and the test conditions in mind, and based on the best published data. Wilson and Fawcett [14] used values of the friction coefficient in the range of 0.007-0.01 and of the restitution coefficient in the range of 0.4-0.6, to investigate the dynamics of the slider-crank mechanism with clearance in the sliding bearing. They showed that the values of friction and restitution coefficients equal to 0.01 and 0.4, respectively, agreed well with experimental data response. Soong [2] developed an experimental device to quantify the restitution coefficient in the impact between a journal and a bearing. Based on the kinematic, or Newton restitution coefficient, Soong obtained 0.46 for the restitution coefficient as the quotient between the impact velocity (5.81 cm/s) and the rebound velocity (2.67 cm/s). In addition, Soong used the Coulomb's friction law to obtain the friction coefficient. Thus, in the present work the values of 0.01 and 0.46 respectively for friction and restitution coefficients are used to model the experimental slider-crank mechanism.

The differences between the experimental and numerical system response profiles, plotted in Figures 5 and 6, must be considered in light of the assumptions made in the formulation for the mathematical model of clearance joints, namely in what concerns to the joints flexibility, which was ignored. The friction in the sliding bearing was also neglected.

Moreover, during the non-contact situation between journal and bearing, in the experimental model the slider velocity is not constant but decreases due to the friction in the sliding bearing, because a preloaded linear bearing type was used. For the numerical simulation, when the journal and bearing do not contact each other, the slider velocity is obviously constant and, consequently, the slider acceleration is null. This phenomenon is well visible as horizontal lines in slider acceleration diagrams of Figures 5f and 5h.

Furthermore, misalignment between journal and bearing elements, always present in actual mechanical systems, were not considered in the numerical simulations. Another important feature in the numerical simulations is the choice of the restitution and friction coefficients, because small variations on these parameters can significantly change the system response. The friction and restitution coefficients depend on the materials and geometric properties of the contacting bodies and can vary during the contact. However, the details of these phenomena were considered beyond the scope of the present work and the choice of the restitution and friction coefficients were based on the best published data.

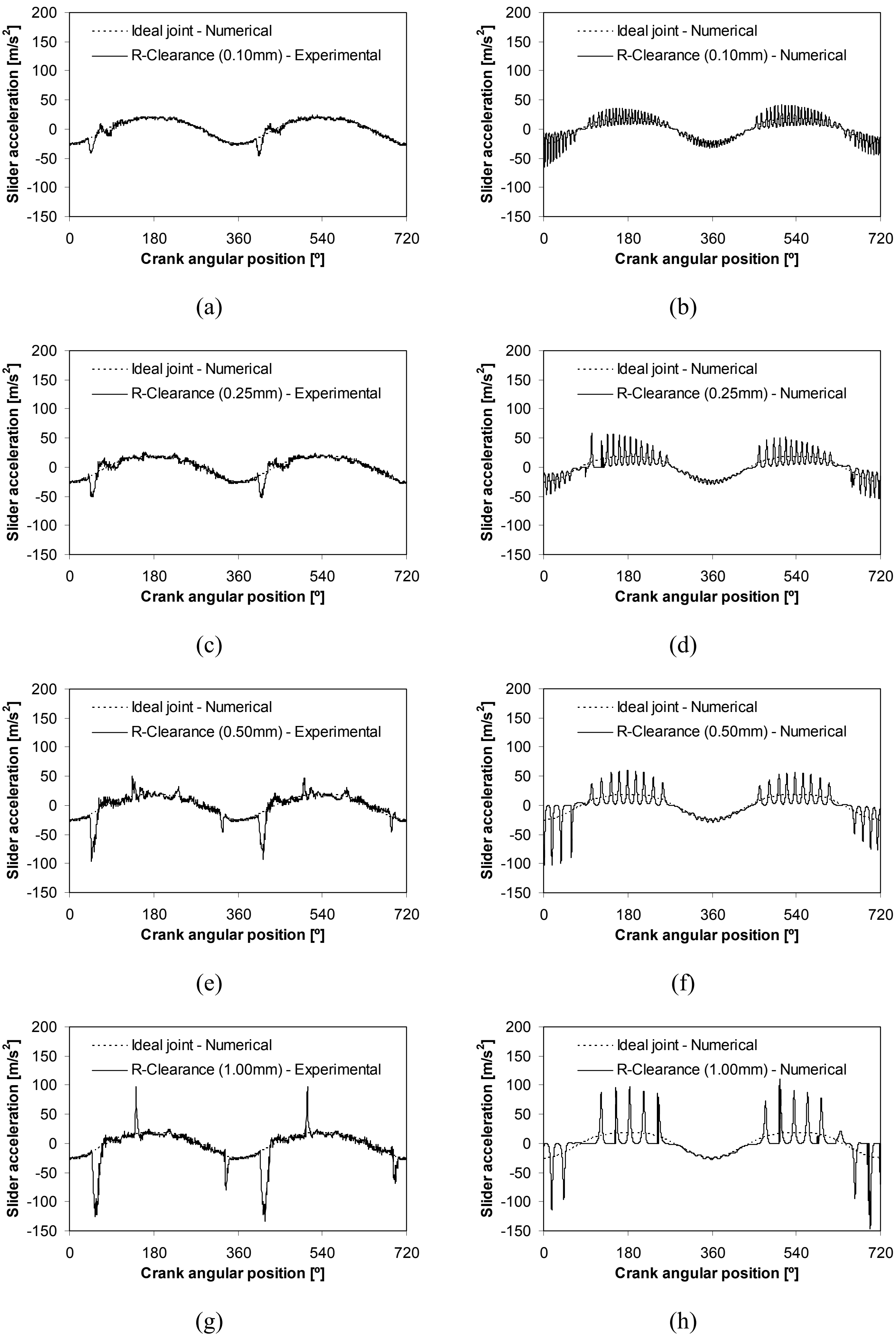


Figure 5 – Experimental and numerical slider acceleration for crank speed of 200 rpm and different radial clearance sizes: (a)-(b) c=0.10 mm; (c)-(d) c=0.25 mm; (e)-(f) c=0.50 mm; (g)-(h) c=1.00mm.

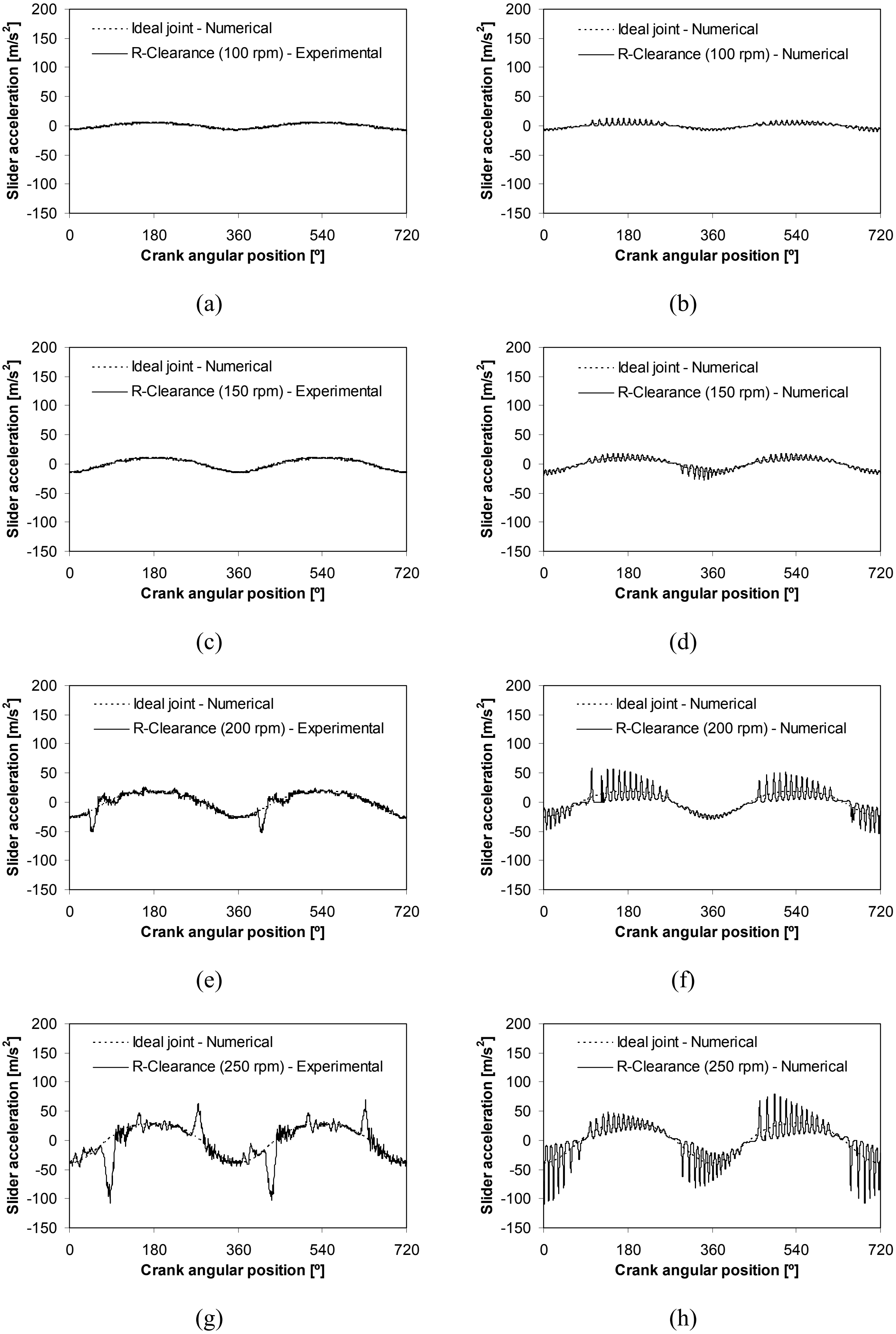


Figure 6 – Experimental and numerical slider acceleration for different crank speeds and clearance size equal to 0.25 mm: (a) 100 rpm; (b) 250 rpm; (c) 200 rpm; (d) 250 rpm.

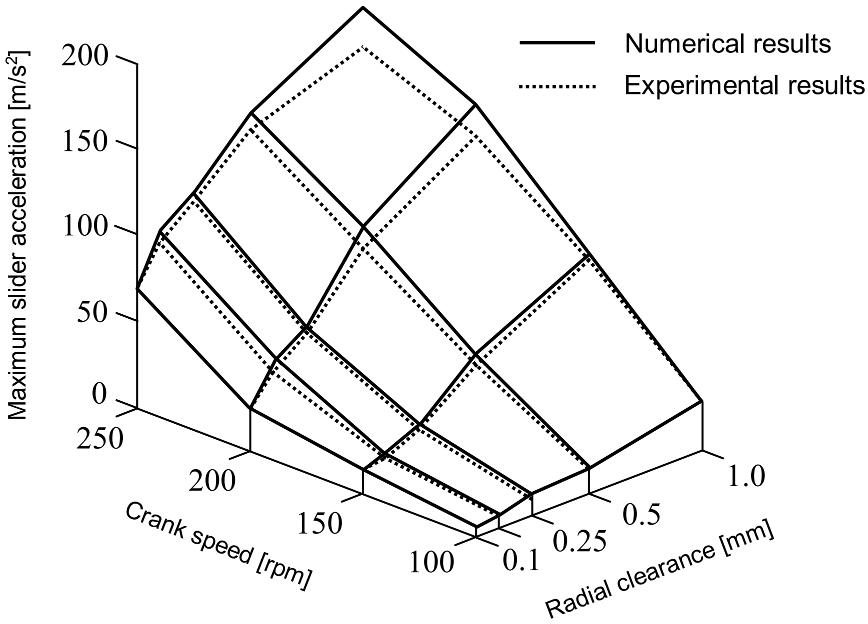


Figure 7 – Maximum slider acceleration as function of crank speed and radial clearance: numerical and experimental results.

Figure 7 presents the maximum slider acceleration, for both experimental and numerical dynamic response histories, as function of crank speed for various radial clearances. From Figure 7, it is clear that the maximum slider acceleration increases with crank speed and with clearance, for both experimental and numerical results. Moreover, the maximum acceleration obtained from experimental tests agrees quite well with numerical simulations, suggesting that the predictive capability of the proposed methodology is a reasonable approach to model multibody systems with impact bodies. This observation confirms other data published on the field on dynamics of multibody systems with clearance joints [2]. The fact that the existence of the clearance joint has an important effect on the slider acceleration supports the idea that the model of clearance joints must be considered in the analysis and design of the real mechanical systems.

CONCLUDING REMARKS

In this work a comprehensive investigation was undertaken for a slider-crank mechanism in which the revolute joint between the slider and the connecting rod had a controlled clearance. Dynamic response data for different operating frequencies, i.e., various crank speeds, with different radial clearance were presented. Prior to present the dynamic response histories with clearance joints, the experimental model was used to obtain the global motion characteristics of the slider-crank mechanism been used joints as close to ideal joints as possible, i.e., with very low clearance ratio. The experimental response data was then compared with numerical results.

The dynamic behavior of the experimental slider-crank setup was monitored with an LVDT and an accelerometer. The response histories were related to the crank angle using an encoder. For comparative purposes, the maximum amplitude of the slider acceleration was taken as a measure of the severity of the impact. This approach provided a good way of estimating the impact force, since with the actual test rig apparatus the direct quantification of the impact force is not possible. It was observed that the maximum amplitude of the impact acceleration increased with clearance and crank speed. For low crank speeds, the gross motion characteristics of the slider-crank mechanism remains similar to those of the mechanism with perfect joints. However, for high frequencies and clearances the dynamic response is significantly altered with the maximum impact acceleration increasing by a factor of more than 20 for this slider-crank. This information is important in the design of mechanical systems and allows to estimate the tradeoff between journal-bearing life, cost and performance. In addition, the information relative to the maximum impact acceleration can be used to help the control and maintenance of industrial machines with clearance joints.

Some critical parameters and test uncertainties should be remarked here. The variety of material joint properties, mainly friction and restitution coefficients, should be considered since the selection of these parameters influences the outcome of the numerical results. In absence of better reference, values published in the literature were used. The correct alignment of the impacting bodies is quite difficult to obtain experimentally, thus, the contact force experienced by these may not be distributed over the theoretical contact area. The methodology used in this work is based on some simplifications that include: the contact surfaces are considered as ellipsoidal; the center of the contact area does not change during the contact process; all the components of the system are rigid bodies.

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